The flavor problem and discrete symmetries

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Abstract

In this Letter we propose a multi-Higgs extension of the standard model with Abelian and non-Abelian discrete symmetries in which the mass matrices of the charged fermions obtained from renormalizable interactions are diagonal. However, non-diagonal contributions, that are important for obtaining the CKM matrix in the quark sector, arise from non-renormalizable dimension five interactions. Active neutrinos acquire mass only from non-renormalizable interactions, the non-diagonal entries arising through dimension five operators, while the diagonal entries comes from dimension six operators. Realistic mixing matrices in the neutrino and the quarks sectors are obtained.

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The flavor problem can be divided in several subproblems [1]: Why do weak isospin partners have different masses? Why are quark and lepton masses split? Why is there a mass hierarchy between generations? Presently we know that neutrino oscillation data are well described by massive neutrinos [2] and the flavor problem becomes more interesting: why is there a mixing angle hierarchy in the quark sector but not in the lepton sector? Usually mass matrices have the form $M_{\alpha\beta} = \sum_i (\Gamma_i)_{\alpha\beta} \langle \Phi_i^0 \rangle$, where the Γ_i's are, for Dirac fermions, arbitrary complex dimensionless 3×3 matrices, and $\langle \Phi_i^0 \rangle$ denotes one or a set of vacuum expectation values (VEVs) of the neutral scalar fields in the model. For Majorana fermions, the Γ_i's are complex symmetric matrices. The mixing matrix and the mass pattern of each charged sector depend on the structure of the respective Γ_i's. It is well known that explicit and predictive

forms of these matrices can be obtained by imposing flavor symmetries. For instance, it has been considered global unitary (horizontal) symmetries like $U(1)$'s, $SU(2)_H$ [3] or $SU(3)_H$ [4]. More recently symmetries like S_n, Z_n [5]; A_4 [6], $^{(d)}T$ [7,8], the double covering of A_4; Q_6, the double dihedral group [9], and other discrete symmetries [10] have been also considered.

Here we will turn the problem upside down. Mass matrices will be of the form $M_{\alpha\beta} = f(\overline{\langle\Phi^0\rangle})_{\alpha\beta}$ in which f denotes a few dimensionless $(\mathcal{O}(1))$ parameters (one at leading order), and $\overline{\langle\Phi^0\rangle}$ is a matrix built with the VEVs of several scalar fields. At first sight, there is no gain in predictive power, we are just changing a dimensionless general matrix $\Gamma_{\alpha\beta}$ by another one with mass dimension $(\overline{\langle\Phi^0\rangle})_{\alpha\beta}$. However, it seems easier, at least in principle, to explain patterns of dynamical variables like VEVs, than dimensionless numbers. The value of the former can be explained by the dynamics (for instance by studying the scalar potential) and extra flavor symmetries that we can impose to the model.

In this Letter we will consider an extension of the standard model in which the charged fermion masses, that arise from renormalizable interactions, are diagonal. However, dimension 5 operators [11] give important non-diagonal contributions to quarks but not to the charged leptons. On the other hand, the neutrinos mass matrix is generated mainly from dimension 5 and 6 operators, with the former having zero trace as in the Zee's mode [12]. The dimension 6 operators induce non-negligible diagonal terms and a general 3×3 symmetric mass matrix for those neutrinos is obtained. The non-diagonal elements are important in the quark sector because they involve also the VEV of the singlet which it is not constrained by the electroweak scale, and also in the neutrino sector because theses entries involve ratios of doublets' VEVs that produce an enhancement. This enhancement does not occurs in the charged lepton sector because in this case the non-diagonal entries depend on the same doublet's VEVs that appear in the diagonal ones but are suppressed by Λ^{-1}. No right-handed neutrinos are introduced. In summary, we show that all the questions mentioned above have a natural answer: fermion masses and mixing arise from different scalar sectors and from different effective interactions. The predictive power is a consequence of the discrete symmetries imposed to the model.

Simple non-Abelian discrete groups have been considered for the first time as candidates for flavor symmetries by Frampton and Kephart [7]. Later, A_4 symmetry has been used to obtain realistic mixing matrices in the lepton sector [13], and then also in the quark sectors [14]. More recently [15], by using these non-Abelian discrete symmetries it was possible to obtain a tribimaximal neutrino mixing matrix [16]. The basic idea is to consider $SU(2)_L$ fermion doublets and singlets in the triplet representation of the A_4 symmetry and Higgs multiplets transforming as triplet or singlet of A_4.

Let us briefly review some properties of the A_4 representations. Denoting the vector basis ($\mathbf{3}$) as (a_1, a_2, a_3) and (b_1, b_2, b_3) we have $\mathbf{3} \otimes \mathbf{3} = \mathbf{3}_A \oplus \mathbf{3}_B \oplus \mathbf{1} \oplus \mathbf{1}' \oplus \mathbf{1}''$. Explicitly [17],

$$\begin{aligned}
[\mathbf{3} \otimes \mathbf{3}]_A &\equiv \mathbf{3}_A = (a_2 b_3, a_3 b_1, a_1 b_2), \quad [\mathbf{3} \otimes \mathbf{3}]_B \equiv \mathbf{3}_B = (a_3 b_2, a_1 b_3, a_2 b_1), \\
[\mathbf{3} \otimes \mathbf{3}]_1 &= a_1 b_1 + a_2 b_2 + a_3 b_3, \qquad [\mathbf{3} \otimes \mathbf{3}]_{1'} = a_1 b_1 + \omega a_2 b_2 + \omega^2 a_3 b_3, \\
[\mathbf{3} \otimes \mathbf{3}]_{1''} &= a_1 b_1 + \omega^2 a_2 b_2 + \omega a_3 b_3,
\end{aligned} \tag{1}$$

and $\mathbf{1} \otimes \mathbf{1} = \mathbf{1}$, $\mathbf{1}' \otimes \mathbf{1}'' = \mathbf{1}$, $\mathbf{1}' \otimes \mathbf{1}' = \mathbf{1}''$, $\mathbf{1}'' \otimes \mathbf{1}'' = \mathbf{1}'$.

We consider a model with $G_{SM} \otimes A_4 \otimes Z_3 \otimes Z_3' \otimes Z_3''$ symmetry, where G_{SM} is the gauge symmetry $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$. The matter fields transform as usual under G_{SM}, left-handed lepton and quark doublets L_a, $a = e, \mu, \tau$; Q_i, $i = 1, 2, 3$, and their respective right-handed singlets l_{aR}, u_{iR} and d_{iR}. In the scalar sector, we introduce scalar doublets with $Y = +1$: H_i, $\hat{H}_i$, H_i', H_i'', Φ_i, Φ_i', Φ_i'' and χ; three scalar triplets with $Y = +2$, denoted by $\mathcal{T}_i$ [18], and finally one complex (with $Y = 0$) scalar singlet ζ. Fermion weak eigenstates fields transform under (A_4, Z_3, Z_3', Z_3'') as follows:

$$\begin{aligned}
L &\equiv (L_e, L_\mu, L_\tau) \sim (\mathbf{3}, \omega, \omega^2, 1); \qquad l_R \equiv (\mu_R, \tau_R, e_R) \sim (\mathbf{3}, 1, 1, \omega), \\
Q_L &\equiv (Q_{1L}, Q_{2L}, Q_{3L}) \sim (\mathbf{3}, \omega^2, 1, 1); \qquad u_R \equiv (c_R, t_R, u_R) \sim (\mathbf{3}, 1, 1, \omega); \\
d_R &\equiv (s_R, b_R, d_R) \sim (\mathbf{3}, 1, \omega, \omega),
\end{aligned} \tag{2}$$

in which $\omega = e^{2\pi i/3}$. Notice that all fermion fields are still symmetry eigenstates but we do not use an special notation. On the other hand, scalar fields transform under (A_4, Z_3, Z_3', Z_3'') as:

$$\begin{aligned}
H &\equiv (H_1, H_2, H_3) \sim (\mathbf{3}, \omega, \omega^2, 1), \quad \hat{H} \equiv (\hat{H}_1, \hat{H}_2, \hat{H}_3) \sim (\mathbf{3}, \omega, \omega^2, \omega^2), \\
H' &\equiv (H_1', H_2', H_3') \sim (\mathbf{3}, \omega, 1, \omega), \quad H'' \equiv (H_1'', H_2'', H_3'') \sim (\mathbf{3}, \omega^2, \omega^2, \omega^2), \\
\Phi &\equiv (\Phi_1, \Phi_2, \Phi_3) \sim (\mathbf{3}, 1, 1, 1), \quad \Phi' \equiv (\Phi_1', \Phi_2', \Phi_3') \sim (\mathbf{3}, \omega, \omega, \omega), \\
\Phi'' &\equiv (\Phi_1'', \Phi_2'', \Phi_3'') \sim (\mathbf{3}, \omega^2, 1, \omega^2), \quad \chi \sim (\mathbf{1}, \omega^2, \omega, 1), \\
\mathcal{T} &\equiv (\mathcal{T}_1, \mathcal{T}_2, \mathcal{T}_3) \sim (\mathbf{3}, \omega^2, \omega, 1), \quad \zeta \sim (\mathbf{1}, 1, \omega, 1).
\end{aligned} \tag{3}$$

With the fields in (2) and (3) we have the leading contributions to the lepton and quark Yukawa interactions:

$$\begin{aligned}
\mathcal{L} =& \frac{1}{\Lambda} \left(f_\nu \, [(\overline{L^c} \epsilon H)]_A [(L\epsilon\Phi)]_B + \frac{f_\nu'}{\Lambda} [\overline{L^c} \varepsilon \vec{\sigma} \cdot \vec{\mathcal{T}}]_B [L\Phi^\dagger]_B \chi + \dots \right) + H.c. \\
&+ \left(g_l [\overline{L}\hat{H}]_A + \frac{g_l'}{\Lambda^2} [\overline{L}\hat{H}]_A |\zeta|^2 + \cdots \right) l_R + H.c.
\end{aligned}$$

$$+ \left(h_u [\overline{Q_L} \widetilde{H'}]_A + \frac{h'_u}{\Lambda} [\overline{Q_L} \widetilde{\Phi'}]_B \zeta + \cdots \right) u_R + H.c.$$

$$+ \left(h_d [\overline{Q_L} H'']_A + \frac{h'_d}{\Lambda} [\overline{Q_L} \Phi'']_B \zeta^* + \cdots \right) d_R + H.c., \tag{4}$$

where Λ is an energy scale characterizing an unknown physics; and $\varepsilon = i\sigma_2$ (σ_2 being the Pauli matrix), $[\ ]_{A,B}$ means the appropriate product defined in Eq. (1) and $\cdots$ denotes higher-dimensional operators, we assume that $|V| \ll \Lambda$, where V denotes the VEV of any of the scalar of the model.

The contributions to the quark mass matrices obtained from renormalizable interactions are diagonal, but the non-renormalizable dimension 5 interactions contribute with important and not negligible non-diagonal elements, on the other hand, for the charged leptons this corrections come from dimension 6 operators are negligible. We obtain diagonal entries to the mass matrices as a consequence of the definition of the basis in which the product $[[\ ]_A \cdot]_1$ gives the diagonal contribution while the product $[[\ ]_B \cdot]_1$ gives the non-diagonal contributions to the mass matrix. For instance, for the u-quarks, the leading diagonal mass matrix arises through the interaction $[\bar{Q}H']_A = (\bar{Q}_2 H'_3, \bar{Q}_3 H'_1, \bar{Q}_1 H'_2)$ and then $[[\bar{Q}H']_A(c_R, t_R, u_R)]_1 = \bar{Q}_2 H'_3 c_R + \bar{Q}_3 H'_1 t_R + \bar{Q}_1 H'_2 u_R$. Similarly, using the $[\ ]_B$ product, we obtain that $[[\bar{Q}\Phi']_B(c_R, t_R, u_R)]_1 = \bar{Q}_3 \Phi'_2 c_R + \bar{Q}_1 \Phi'_3 t_R + \bar{Q}_2 \Phi'_1 u_R$, which gives the non-diagonal contributions to the charged lepton mass matrix.

Hence, from the Yukawa interactions in Eq. (4), the mass matrices obtained are

$$M^l \approx g_l \begin{pmatrix} \hat{v}_2 & 0 & 0 \\ 0 & \hat{v}_3 & 0 \\ 0 & 0 & \hat{v}_1 \end{pmatrix} + H.c., \tag{5}$$

for the charged leptons. We have denoted $\langle \hat{H}_i \rangle = \hat{v}_i$. Notice that the renormalizable interactions are the dominant one, i.e., $g_l \hat{v}_1 \simeq m_\tau$, $g_l \hat{v}_3 \simeq m_\mu$ and $g_l \hat{v}_2 \simeq m_e$. In the quark sector we have

$$M^u \approx h_u \begin{pmatrix} v'_2 & 0 & a_u v'_{\phi_3} \\ a_u v'_{\phi_1} & v'_3 & 0 \\ 0 & a_u v'_{\phi_2} & v'_1 \end{pmatrix} + H.c., \tag{6}$$

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and

$$M^d \approx h_d \begin{pmatrix} v_2'' & 0 & a_d v_{\phi_3}'' \\ a_d v_{\phi_1}'' & v_3'' & 0 \\ 0 & a_d v_{\phi_2}'' & v_1'' \end{pmatrix} + H.c., \tag{7}$$

in which $a_u = \frac{h_u'}{h_u}\frac{v_\zeta}{\Lambda}$ and $a_d = \frac{h_d'}{h_d}\frac{v_\zeta}{\Lambda}$, for the 2/3 and $-1/3$ charged quarks, respectively. We have denoted $\langle H_i^0 \rangle = v_i$, $\langle H_i'^0 \rangle = v_i'$, $\langle H_i''^0 \rangle = v_i''$, $\langle \Phi_i \rangle = v_{\phi_i}$, $\langle \Phi_i' \rangle = v_{\phi_i}'$, $\langle \Phi_i'' \rangle = v_{\phi_i}''$, and $\langle \zeta \rangle = v_\zeta$. Notice, as well, that each charged sector has its private VEVs, thus all these matrices are independent from each other. In the case of neutrinos we have the mass matrix

$$M^\nu \approx \left[\begin{pmatrix} 0 & \frac{v_2}{v_1} & \frac{v_{\phi_3}}{v_{\phi_1}} \\ \frac{v_2}{v_1} & 0 & \frac{v_3}{v_1}\frac{v_{\phi_2}}{v_{\phi_1}} \\ \frac{v_{\phi_3}}{v_{\phi_1}} & \frac{v_3}{v_1}\frac{v_{\phi_2}}{v_{\phi_1}} & 0 \end{pmatrix} + \begin{pmatrix} \delta_1 & 0 & 0 \\ 0 & \delta_2 & 0 \\ 0 & 0 & \delta_3 \end{pmatrix} \right] \frac{f_\nu v_{\phi_1}}{\Lambda} v_1, \tag{8}$$

where the δ_i are given by $\delta_1 = \frac{f_\nu'}{f_\nu}(v_{T3}v_{\phi_3}v_\chi/\Lambda v_1 v_{\phi_1})$, $\delta_2 = \frac{f_\nu'}{f_\nu}(v_{T1}v_\chi/\Lambda v_1)$, and $\delta_3 = \frac{f_\nu'}{f_\nu}(v_{T2}v_{\phi_2}v_\chi/\Lambda v_1 v_{\phi_1})$, where $\langle \chi \rangle = v_\chi$, and $\langle \Delta_i^0 \rangle = v_{Ti}$.

It is interesting that the discrete symmetries of the model implies that the charged lepton mass matrix, Eq. (5), is practically diagonal, and that the quark mass matrices have three zeros, see Eqs. (6) and (7). To the best of our knowledge, this mass matrices have not been considered in literature [19]. Notice that in the case of the neutrino we have already taken into account the hermitian conjugate.

Since the only constraint on the VEVs of the doublets is that $\sum v^2 < (174 \text{GeV})^2$ (there are triplets as well, see below), a possibility is that only $v_1'^2 \lesssim (174\,\text{GeV})^2$, while all the other VEVs are $\ll 174$ GeV. In this case v_1' give the main contribution to the mass of the t quark. On the other hand, the sum of the VEVs of the triplets has to have a upper limit of the order of a few GeV to not spoil the observed value of the ρ parameter [20]. Hence, $|V|/\Lambda$ are well-defined expansion parameters. The present model is natural in the Yukawa interactions i.e., unlike the standard model, there is no a hierarchy among the Yukawa couplings. In practice, in the analysis below we will consider that all Yukawa coupling are in the interval $0.1 - 3$. The VEV of the singlet v_ζ is only constrained by the condition $v_\zeta/\Lambda < 1$. Below we will use always, just as an illustration: $\Lambda = 1$ TeV and $v_\zeta = 140$ GeV. However, recall that the singlet ζ may be related to a new energy scale different from the electroweak scale.

As a first illustration, let us consider the matrix in Eq. (8). We will as-

sume, without losing generality, that this matrix is a traceless matrix i.e., that the condition $\sum_i \delta_i = 0$ is valid [12]. This is obtained by imposing (using $\tilde{v}_x = v_x e^{i\theta_x}$, where v_x and θ_x are real):

$$\theta_{T3} + \theta_{\phi_3} - \theta_1 - \theta_{\phi_1} = \pi, \quad \theta_{T1} - \theta_1 = 0; \quad \theta_{T2} + \theta_{\phi_1} - \theta_1 - \theta_{\phi_1} = 0,$$

$$(1/2)\left|\frac{f'_\nu}{f_\nu} \frac{v_{T3} v_\chi v_{\phi_3}}{\Lambda v_1 v_{\phi_1}}\right| = \left|\frac{f'_\nu}{f_\nu} \frac{v_{T1} v_\chi}{\Lambda v_1}\right| = \left|\frac{f'_\nu}{f_\nu} \frac{v_{T2} v_\chi v_{\phi_2}}{\Lambda v_1 v_{\phi_1}}\right| \equiv \delta. \tag{9}$$

Hence, the neutrino mass matrix is of the form [21,22]

$$M^\nu \approx m_0 \begin{pmatrix} -2\delta & \sin\theta & \cos\theta \\ \sin\theta & \delta & \epsilon \\ \cos\theta & \epsilon & \delta \end{pmatrix}, \tag{10}$$

where $m_0 = (f_\nu/\Lambda)|v_{\phi_1} v_1|$, $|v_2/v_1| = \sin\theta$, $|v_{\phi_3}/v_{\phi_1}| = \cos\theta$, $\epsilon = |(v_3 v_{\phi_2}/v_1 v_{\phi_1})|$. From Table I of Ref. [22] we can see that neutrinos masses and mixing angles are in agreement with all experimental data if, for instance, in Eq. (10), $m_0^2 = \Delta m_{13}^2 \equiv \Delta m_{atm}^2 = (0.039 \, \text{eV})^2$, $\epsilon = 0.21$, $\delta = 0.236$, and $\theta = 0.84$. In our model, these values for the above parameters are obtained by assuming: $\Lambda = 1$ TeV, $v_1 \sim 1$ MeV, $v_{\phi_1} = v_{\phi_2}$, $2v_{T1} = 2v_{T2} = v_{T3}$. with $v_{T1} = 1.25$ GeV; then we obtain: $f_\nu v_{\phi_1} \sim 40$ keV, $v_{\phi_3} = 40/f_\nu$ keV, $v_2 = 0.1$ keV, $v_3 = 0.2$ MeV, $(f_\nu/f'_\nu)v_\chi = 0.2$ GeV. We also obtain with these VEVs, $\Delta m_{12}^2 = 9 \times 10^{-5}$ eV^2, $\sin^2 2\theta_\odot = 0.82$, $\sin^2 2\theta_{atm} = 0.99$, and $\sin\theta_{13} = 0.019$ a value that is in agreement with the CHOOZ limit $V_{e3} = \sin\theta_{13} < 0.07$ [23]. We see that a realistic leptonic mixing matrix arises since the mass matrix of the charged leptons is almost diagonal. If $f_\nu, f'_\nu \sim \mathcal{O}(1)$, it implies that some VEVs are of the order of MeV or even less depending on the values of v_ζ and Λ. This does not necessarily implies the existence of light scalars since for having them in a model, there must be some symmetries that allow them to appear in the mass spectra: The symmetries in the present model do not play that role and μ_{ij}^2 terms survive in the scalar mass matrices (see Ref. [24]).

On the other hand, with $M^{u,d}$ given by Eq. (6) and (7) we see that $M^{u(d)} M^{u(d)\dagger} \neq M^{u(d)\dagger} M^{u(d)}$. Hence we need four matrices, $V_{L,R}^{U,D}$, for diagonalizing these mass matrices by biunitary transformations and the constraint $V_{CKM} = V_L^U V_L^{D\dagger}$. Besides, the matrices $V_{L,R}^{U,D}$ must satisfy the constraints $V_L^D M^d M^{d\dagger} V_L^{D\dagger} = diag(m_d^2, m_s^2, m_b^2) = (\hat{M}^d)^2$ and $V_R^D M^{d\dagger} M^d V_R^{D\dagger} = (\hat{M}^d)^2$; similarly for the $V_{L,R}^U$ matrices. The quark masses and the V_{CKM} matrix arise, for instance, using for the d-type quarks $a_d = 0.14$, $h_d = h'_d = 0.1$, and (all VEVs and masses below are in GeV) $v''_1 = 42$, $v''_2 = 1.4$, $v''_3 = 0.062$, $v''_{\phi_1} = 2.552$, $v''_{\phi_2} = 0.0074$, and $v''_{\phi_3} = 11.83$ we obtain the masses: $m_d = 0.006$, $m_s = 0.1$, $m_b = 4.2$. For the u-type quarks we use, $a_u = a_d$, but now $h_u = h'_u = 3$, and $v'_1 = $

53.8, $v_2' = 0.4$, $v_3' = 0.0001$ and $v_{\phi_1}' = 0.1$, $v_{\phi_2}' = 124$, $v_{\phi_3}' = 0.082$, which give $m_u = 0.0025$, $m_c = 1.27$, and $m_t = 171.2$. With these VEVs and parameters we obtain $V_L^{D,U}$ matrices given, according with the definition above, the mixing matrix

$$|V_{CKM}| = \begin{pmatrix} 0.97747 & 0.21105 & 0.00148 \\ 0.21083 & 0.97673 & 0.03923 \\ 0.00972 & 0.03803 & 0.99923 \end{pmatrix}. \tag{11}$$

Comparing the elements of the matrix above with experimental data in Section 11.2 of PDG [25], we see that all the matrix elements, but us and ub entries, are within the level of confidence (1-2)σ. However, this is enough to show that a realistic mixing matrix may arise from the quark mass matrices given in Eqs. (6) and (7). A more rigorous and detailed analysis is being done and will be presented later.

The suppression of the FCNCs coupled to neutral Higgs bosons can be obtained not naturally in the sense of Ref. [26], but at least as a reasonable fine tuning among the mixing matrix in the neutral scalar sector, U_{ij}, or by the mass of heavy Higgs scalars. For instance, consider the neutral kaons mass difference, $\Delta M_K \sim \zeta_{sd}^i f_K^2 m_K (\sum_j |U_{ij}|^2/m_j^2)$ where $\zeta_{sd}^i = h_d^2 |\sum_a (V_L^D)_{as}^* (V_R^D)_{ad}|^2$, with symmetry eigenstates, say H_i'' (i fixed), and physical scalars h_j with mass m_j. In order to be consistent with data ζ_{sd}^i must be smaller than the contribution of the SM: i.e., smaller that $G_F^2 m_c^2 (V_{cd}^* V_{cs})^2/16\pi^2 = 0.5 \times 10^{-8} \mathrm{GeV}^{-2}$ (for simplicity we have taken into account only the c quark contribution and neglect the QCD and m_c^2/M_W^2 corrections). Notice that the sum over the contributions of each i is made coherently. The expression above is valid for any scalar heavier than m_K. For instance, $h_d^2 |\sum_a (V_L^D)_{as}^* (V_R^D)_{ad}|^2 \sim 10^{-4} - 10^{-6}$ and $|U_{ij}|^2/m_j^2 \sim (10^{-2} - 10^{-4}) \mathrm{GeV}^{-2}$ we have the appropriate FCNC suppression. These constraints are not a large fine tunings since it can be satisfied with a large set of unitary matrices and/or the neutral scalar mass spectrum and mixing. Recall that there is no more freedom to redefine the quark phases since they all have already been used to put the CKM matrix in the usual form, hence V_R^D has five physical phases. The case of FCNCs in the u-quark sector involve the matrices V_L^U and V_R^U.

On the other hand, it would be interesting if we can show that appropriate hierarchies among the VEVs arises from an analysis of the scalar potential [27]. We have verified that, under reasonable assumptions on the parameters in the scalar potential, this is indeed the case. For the sake of simplicity we will consider the scalar potential involving only the scalar doublets that couple to quarks: the A_4 triplets H', Φ' for u-like quarks, and H'', Φ'' for d-like quarks. Denoting $X_u = H'^\dagger H'$ and $Y_u = \Phi'^\dagger \Phi'$, $X_d = H''^\dagger H''$ and $Y_d = \Phi''^\dagger \Phi''$ we

write the most general scalar potential invariant under the SM and $A_4 \otimes Z_3 \otimes Z_3' \otimes Z_3''$ symmetries as:

$$\begin{aligned}
\mathcal{V} = &\ \mu_p^2[X_p]_1 + \mu_p'^2[Y_p]_1 + \mu_\zeta^2|\zeta|^2 + \lambda_\zeta|\zeta|^4 + (\mu_{H'\Phi''}^2[H'\epsilon\Phi'']_1 + \mu_{H''\Phi'}^2[H''\varepsilon\Phi']_1 \\
&+ f[\Phi'^\dagger H']_1\zeta + f'[\Phi''^\dagger H'']_1\zeta^* + f_H'[H''\varepsilon H']_1\zeta + f_\Phi'[\Phi''\epsilon\Phi']_1\zeta^* \\
&+ \lambda_{mn}^{pp'}[X_p]_m[X_{p'}]_n + K_{kl}^{pp'}[Y_p]_k[Y_{p'}]_l + h_{\alpha\beta}^{pp'}[X_p]_\alpha[Y_{p'}]_\beta + g_{st}[H'\epsilon\Phi'']_s[H'\epsilon\Phi'']_t \\
&+ g_{st}'[H''\varepsilon\Phi']_s[H''\varepsilon\Phi']_t + g_{st}''([H'\epsilon\Phi'']_s[H''\varepsilon\Phi']_t) + H.c.).
\end{aligned} \quad (12)$$

We have omitted summation symbols and used the notation $p, p' = u, d$; $mn = 11, 1'1'', AA$; $kl = 11, 1'1'', BB$; $\alpha\beta = 11, 1'1'', AB$; and $st = 11, 1'1'', AA, BB, AB$. For example, $\lambda_{mn}^{pp'}[X_p]_m[X_{p'}]_n = \lambda_{11}^{pp'}[X_p]_1[X_{p'}]_1 + \lambda_{1'1''}^{pp'}[X_p]_{1'}[X_{p'}]_{1''} + \lambda_{AA}^{pp'}[X_p]_A[X_{p'}]_A + \cdots$, and each term having contributions $p, p' = u, d$. Considering the Eq. (12) we obtain the constraint equations arising from the conditions $\partial\mathcal{V}/\partial v_i'' = 0, \ldots$. For simplicity, in the following analysis it will be also assumed that the VEVs are all real. In this case the condition $1 + \omega + \omega^2 = 0$ implies that in the constraint equations the coupling constants will appear as the sum of some of the coupling constants in Eq. (12). The inclusion of all the scalar fields of the model and also the constraints coming from the condition $\partial^2\mathcal{V}/\partial v_i''\partial v_j'' > 0, \ldots$ will be shown elsewhere. We have obtained the constraint equations under the following conditions: $v_i'' = v_{\phi_i}''$, $\forall i$; and $\lambda_{mn}^{dd} = K_{kl}^{dd}$, $\forall mn; kl$; and verified that, among the solutions to those equations, it is possible to have the following hierarchy: $v_1'' \gg v_3'' \gg v_2''$. However for generating the quark masses, as we have shown above, it is necessary that $v_1'^{(\prime\prime)} \gg v_{\phi_1}'^{(\prime\prime)}$. Notwithstanding, it is interesting that such hierarchies can arise from the model, and we hope that a more detailed treatment of the scalar potential will produce a more realistic hierarchies among the VEVs.

As in any model with broken discrete symmetries there are potential troubles with domain walls [28]. However, this issue can be solved by allowing soft terms in the scalar potential violating these symmetries.

In conclusion, the mass matrices obtained, Eqs. (5), (6), (7), and (8), arising because of the symmetries of the model, give appropriate insight concerning the solution of the flavor problem generating at the same time a realistic V_{CKM} matrix. Of course, it is necessary to explain how these symmetries are realized from a more fundamental theory. Our scheme has some similarities with the "private Higgs" of Porto and Zee [29] in which there is one Higgs per fermion and a general mass matrices. However, we have more than one Higgs per fermion and the mass matrices have a specific texture of zeros.

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